

Properties of few-body systems in relativistic quantum mechanics and constraints from transformations under Poincaré space-time translations

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Different approaches have been applied to the calculation of form factors of various hadronic systems within relativistic quantum mechanics. In a one-body current approximation, they can lead to results evidencing large discrepancies. Looking for an explanation of this spreading, it is shown that, for the largest part, these discrepancies can be related to a violation of Poincaré space-time translation invariance. Beyond energy-momentum conservation, which is generally assumed, fulfilling this symmetry implies specific relations that are generally ignored. Their relevance within the present context is discussed in detail both to explain the differences between predictions and to remove them.

1. INTRODUCTION AND MOTIVATIONS FROM FORM FACTORS

The implementation of relativity within quantum mechanics has been the object of extensive studies these last years, especially with regard to the prediction of properties of various hadronic systems, such as form factors. These ones could be a test for the underlying dynamics but this program supposes that accounting for relativity is under control. Looking at predictions, which are generally based on a single-particle current, it is found that they can considerably differ, depending on the form [1] and the kinematics used for their calculation. The effect is especially large when the mass of the system is small in comparison of the sum of the constituent ones (pion for instance). The study of a theoretical model with a simple dynamics can give insight on the relevance of a given approach. It does not necessarily provide an argument why the results are so spread in some cases. It is the aim of the present contribution to bring the attention on the role, with this respect, of Poincaré space-time translation invariance. This symmetry implies energy-momentum conservation, which is generally assumed, but, within relativistic quantum mechanics (RQM), it also supposes some constraints. These ones are rarely considered and their fulfillment requires that the contribution of many-body currents be included.

In the second section, we remind results obtained in different approaches for the charge form factor of the ground state in a simple theoretical system. The third section is devoted to the role of Poincaré space-time translation invariance. The violation of the expected constraints is related to the discrepancies between various approaches. Results corrected for this violation are presented. Some prospects are given in the fourth section.

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2. FORM FACTORS FROM A SIMPLE THEORETICAL MODEL

We here remind results [2] obtained for the charge form factor of the ground state in the Wick-Cutkosky model. This one consists of scalar particles exchanging a massless scalar meson. It offers the advantage to be solvable and the resulting Bethe-Salpeter amplitude can easily be used to calculate form factors. This represents our “experiment”. The corresponding form factors can be calculated in RQM approaches using the solution of a mass operator. This one is obtained from a fit to the lower-states masses, while fulfilling minimal requirements [3]. The total and constituent masses are chosen as for a pion-like system. The detailed expression of form factors, which can be found in Ref. [2], is common to all cases. It is in particular noticed that the current has a one-body form and assumes the most natural choice, $\langle J^\mu \rangle \propto (p_i + p_f)^\mu$ (with $p^0 = +\sqrt{m^2 + p^2}$).

Numerical results are presented in Fig. 1, at small Q^2 to show the sensitivity to the charge radius and at high Q^2 (multiplied by Q^4) to evidence the asymptotic behavior. The approaches under consideration involve:

- the front form with $q^+ = 0$ (“perpendicular” kinematics: F.F.(perp.)),
- the instant form in the Breit frame (“perpendicular” kinematics: I.F.(Breit frame)),
- the “point-form” (Bakamjian [4], Sokolov [5]: “P.F.”),
- the point-form (Dirac inspired [6]: D.P.F.),
- the front-form (“parallel” kinematics, $(\vec{P}_i + \vec{P}_f) \parallel \vec{Q} \parallel \vec{n}$: F.F.(parallel)),
- the instant-form (“parallel” kinematics, $(\vec{P}_i + \vec{P}_f) \parallel \vec{Q}$, $|\vec{P}_i + \vec{P}_f| \rightarrow \infty$: I.F.(parallel)).

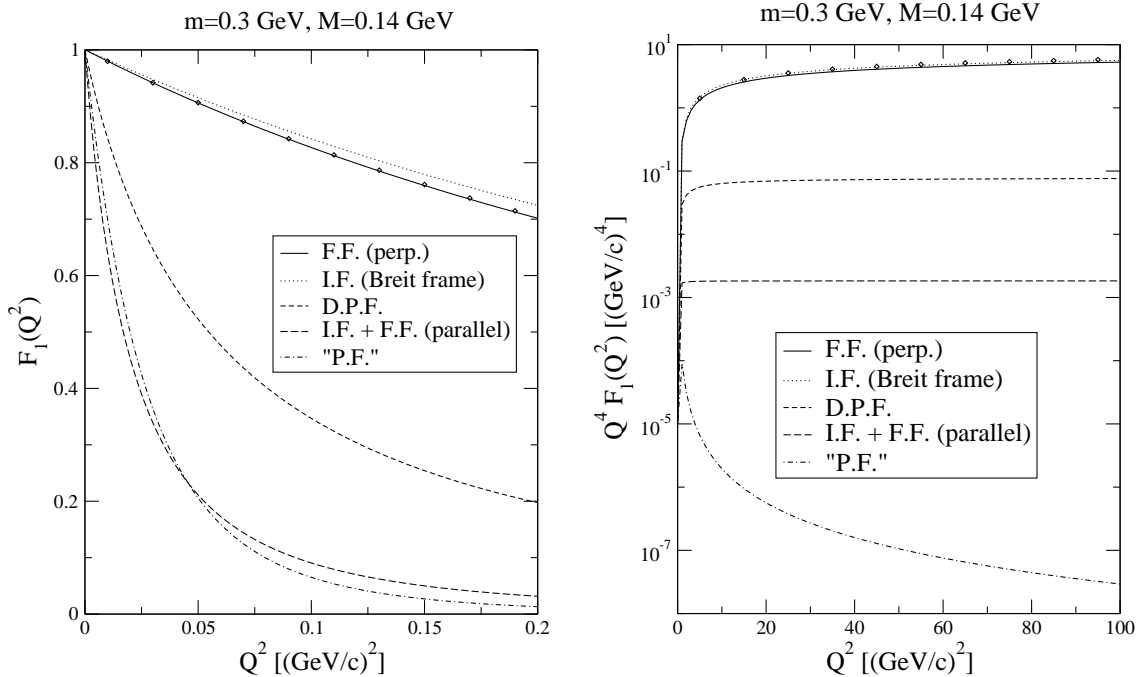


Figure 1. Charge form factor at low and high Q^2 in different forms of relativistic quantum mechanics (see text for details). The “experiment” is represented by small diamonds.

In comparison to the “experiment”, it is noticed that the standard front and instant forms (F.F.(perp.) and I.F.(Breit frame)) do well. In the other cases, a large sensitivity to the smallness of the total mass, M , is observed, the charge radius scaling like M^{-1} . It is also noticed that fulfilling Lorentz invariance (“P.F.” and D.P.F.) does not ensure good results. Altogether, these observations raise a double question: What is the reason for large discrepancies? Why some approaches do better than other ones?

3. CONSTRAINTS FROM POINCARÉ SPACE-TIME TRANSLATION INVARIANCE

Poincaré space-time translation invariance is a symmetry whose consequences for currents in RQM approaches are often ignored, beyond 4-momentum conservation which, of course, is always assumed. Quite generally, it supposes that currents transform as follows:

$$e^{iP \cdot a} J^\nu(x) (S(x)) e^{-iP \cdot a} = J^\nu(x + a) (S(x + a)), \quad (1)$$

where P^μ is the operator of the Poincaré algebra that generates space-time translations. In the particular case $a = -x$, one gets:

$$J^\nu(x) (S(x)) = e^{iP \cdot x} J^\nu(0) (S(0)) e^{-iP \cdot x}. \quad (2)$$

By considering the matrix element of the above current between eigenstates of momenta P_i^μ and P_f^μ and taking into account that these states in RQM approaches are, by construction [7], eigenstates of the operator P^μ , one can factorize the x dependence of the current. When integrating the resulting factor, $\exp(i(P_i - P_f) \cdot x)$, together with the plane wave describing the external probe, $\exp(iq \cdot x)$, one recovers the energy-momentum conservation relation mentioned above, $P_f^\mu - P_i^\mu = q^\mu$. One is left with a matrix element at $x = 0$.

Quite generally however, writing relations given by Eqs. (1) supposes that the currents in RQM approaches, beside a one-body component usually considered, also contain many-body components. In their absence, relativistic covariance cannot be achieved. This can be checked by considering further relations stemming from Eqs. (1) [8]:

$$[P^\mu, J^\nu(x)] = -i\partial^\mu J^\nu(x), \quad [P^\mu, S(x)] = -i\partial^\mu S(x), \quad (3)$$

and especially the double commutator with P^μ , which could be more relevant here:

$$[P_\mu, [P^\mu, J^\nu(x)]] = -\partial_\mu \partial^\mu J^\nu(x), \quad [P_\mu, [P^\mu, S(x)]] = -\partial_\mu \partial^\mu S(x). \quad (4)$$

Considering the matrix element of this last relation, it is found that the double commutator at the l.h.s. can be replaced by the square of the 4-momentum transferred to the system, $(P_i - P_f)^2 = q^2$, while the derivatives at the r.h.s., for a single-particle current, can be replaced by the square of the 4-momentum transferred to the constituents, $(p_i - p_f)^2$ (see Fig. 2 for both a graphical representation and some kinematical notations). A test of Poincaré space-time translation covariance is therefore given by the relation at $x = 0$:

$$< |q^2 J^\nu(0) \text{ (or } S(0))| > = < |(p_i - p_f)^2 J^\nu(0) \text{ (or } S(0))| >. \quad (5)$$

In RQM approaches, the squared momentum transferred to the constituents differs most often from that one transferred to the system: $(p_i - p_f)^2 \neq q^2 (= -Q^2)$. It is therefore

expected that Eq. (5) is not fulfilled. Checking this relation, we found a close relationship between its violation (a factor 3 for D.P.F., 30 for F.F.+I.F.(parallel) and 35000 for “P.F.” at the highest value of Q^2 considered here) and discrepancies between form factors.

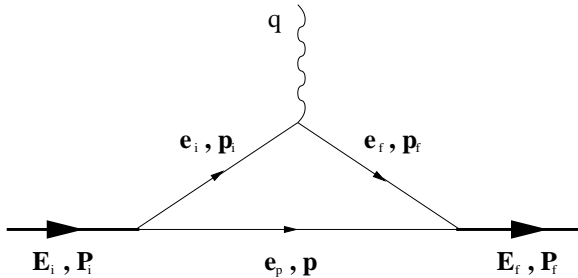


Figure 2. Interaction with an external field together with kinematical notations.

To correct for the violation in a first step, we modified the coefficient of Q in the boost expression so that to fulfill the equality $(p_i - p_f)^2 = q^2$. This can be done analytically in some cases, numerically (on the average) in other ones. The change amounts to account for many-body currents. Interestingly, no modification is required for the standard front form ($q^+ = 0$), where the above equality is always fulfilled. Results so obtained are shown in Fig. 3. It is found that the undesirable dependence of the charge radius on M has vanished and that many orders of magnitude discrepancies at high Q are largely removed.

Concerning a possible extension of present results, we recently found quite similar ones for the charge pion form factor, indicating that the spin of the constituents is not essential in the discussion made here [9]. We also found that remaining discrepancies between different approaches (F.F.(perp.), I.F. (Breit frame), “P.F.” and a dispersion-relation one [10] corrected for some part) could be completely removed by relying on a current that differs from the usual choice. The modification has a small numerical effect but is important in getting the above result. This last achievement supposes a somewhat non-trivial change of variables relating one approach to the other, after accounting for effects from space-time translation invariance as described previously, when analytically possible. Both the numerical results and the recent developments unambiguously show the relevance of correctly implementing Poincaré space-time translation invariance.

4. CONCLUSION AND OUTLOOK

We have shown that large discrepancies between predicted form factors in different RQM approaches could be ascribed to missing properties from Poincaré space-time translation invariance. Relations that could be used to test this symmetry as well as to correct for its violation have been emphasized. Discrepancies are largely removed after this is done. Some of them could be related to a scaling of the charge radius with the inverse of the mass of the system under consideration. This counterintuitive result points to the violation of some symmetry. It is removed when accounting for Poincaré space-time translation invariance. In comparison with Lorentz invariance, whose consequences can

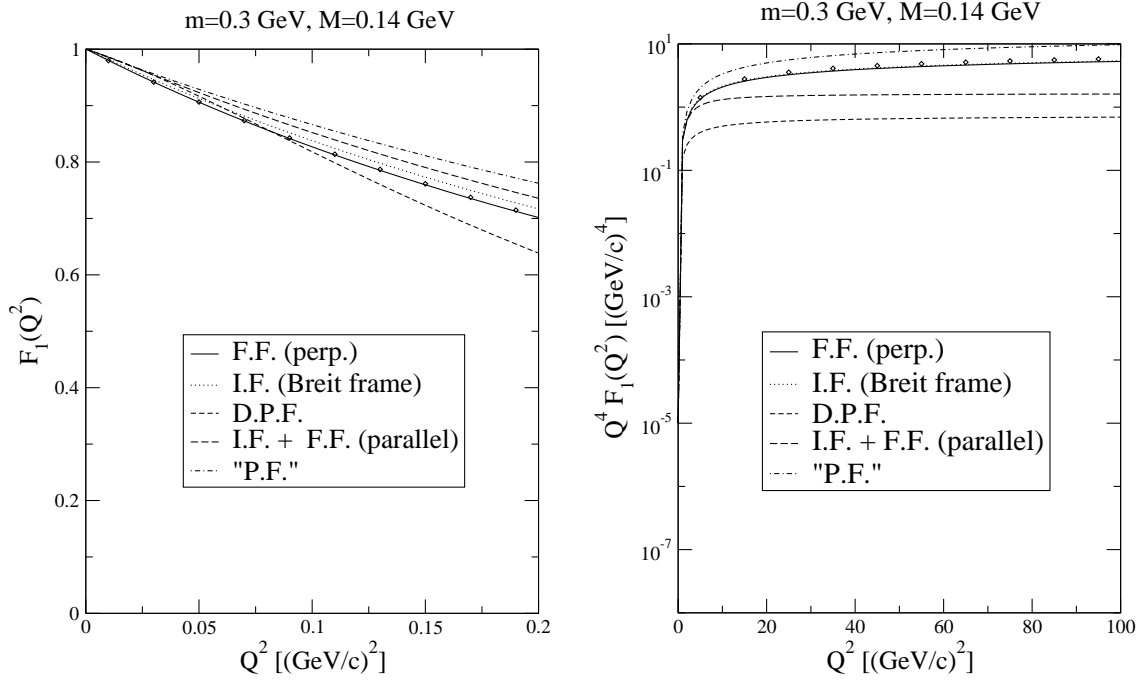


Figure 3. Same as in Fig. 1, corrected for Poincaré space-time translation invariance.

easily be checked, those related to the other symmetry discussed here are more subtle, perhaps explaining why they have not received much attention till now.

Present results point to the standard front-form or instant-form approaches as more efficient ones to implement relativity in describing properties of few-body systems. Various considerations were favoring these approaches but a deep and simple justification was lacking. We believe that fulfilling constraints from Poincaré space-time translation invariance is an essential argument in discriminating between different approaches.

When comparing theoretical predictions to experiment, one could wonder about the reliability of either the approach used to implement relativity or the dynamical ingredients. By reducing the uncertainty about relativity, the present work allows one to concentrate the discussion on the role of the underlying dynamics.

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